Melting Heat Transfer in MHD Boundary Layer Stagnation-Point Flow Towards a Stretching Sheet with Thermal Radiation

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ABSTRACT. An analysis is carried out to study the steady two-dimensional stagnation-point flow and heat transfer from a warm, laminar liquid flow to a melting stretching sheet. The governing partial differential equations are converted into ordinary differential equations by similarity transformation, before being solved numerically using Runge-Kutta-Fehlberg method. Effects of Magnetic parameter, Radiation parameter, melting parameter, stretching parameter and Prandtl number on flow and heat transfer characteristics are thoroughly examined.

*Note a – Parameter of the temperature distributed in stretching surface; \(B_o\) – Externally imposed magnetic field in the y-direction; \(C_p\) – Drag coefficient; \(C_f\) – Local skin-friction coefficient; \(c_p\) – Specific heat of the fluid at constant pressure; \(f\) – Dimensionless stream function; \(k\) – Radiation number; \(L\) – Characteristic length of the flow dynamics; \(N_{ht}\) – Nusselt number; \(Pr\) – Prandtl number; \(Nr\) – Radiation Parameter; \(M_m\) – Magnetic Parameter; \(T\) – Fluid temperature; \(T_a\) – Ambient temperature; \(T_0\) – Reference number; \(T_w\) – Wall temperature; \(u\) – Fluid axial velocity; \(U_o\) – Velocity of horizontal stretching surface; \(U_e\) – Velocity of external flow; \(V\) – Fluid transverse velocity; \(x, y\) – Coordinates along and normal to the vertical stretching surface plate; \(X\) – Dimensionless coordinate along the plate; Greek symbol \(\nu\) – Kinematic viscosity; \(\tau_{yy}\) – Local shear stress; \(\alpha\) – Thermal diffusivity; \(\beta\) – Coefficient of thermal expansion; \(\eta\) – Non-dimensional transformed variable; \(\lambda\) – Dimensionless heat generation/absorption parameter; \(\mu\) – Viscosity of the fluid; \(\sigma\) – Fluid electrical conductivity; \(\psi\) – Stream function; \(\rho\) – Fluid density; \(\theta\) – Dimensionless temperature; Subscripts \(x\) – local; \(w\) – Conditions on the wall; \(o\) – Reference; \(\infty\) – Ambient or free stream condition; ‘ – Differentiation with respect to \(t\).

Introduction. The study of boundary layer flow and heat transfer over a stretching surface is important and has attracted considerable interest of many researchers because of its large number of applications in industry and technology. Few of these applications are materials manufactured by polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, cooling of metallic sheets or electronic chips, and many others. In these cases, the final product of desired characteristics depends on the rate of cooling and the rate of stretching. After the pioneering work by Sakiadis [1], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces [2–10].

Stagnation point flow, describing the fluid motion near the stagnation region of a circular body, exists for both the cases of a fixed or moving body in a fluid. The two dimensional stagnation flow towards a stationary semi-infinite wall was first studied by Hiemenz[11], using a similarity transformation, to reduce the Navier-stokes equations to a nonlinear ordinary differential equation.

Chiam [12] examined the stagnation point flow of viscous fluid towards a linear stretching surface. Stagnation-point flow of power-law fluid over a stretching surface was reported by Mahapatra et al. [13]. They have obtained a numerical solution of the problem by fourth-order Runge-Kutta integration technique. Epstein and Cho [14] analyzed the melting heat transfer of the steady laminar flows over
a flat plate. The steady laminar boundary layer flow and heat transfer from a warm, laminar liquid flow to a melting surface moving parallel to a constant free stream has been studied by Ishak et al. [15]. Researchers [16,17,18] have considered to investigate melting heat transfer effects on boundary layer flow and heat transfer over a stretching sheet with various flow and heat transfer conditions involved.

The aim of the present paper is to study the similarity solutions of stagnation-point flow and the heat transfer from a warm, laminar liquid flow to a melting stretching sheet with the combined effect of magnetic field and thermal radiation. A thorough literature survey show that there is not any published work on melting heat transfer in boundary layer stagnation point flow towards a stretching sheet with the effect of magnetic field and thermal radiation. We believe that the obtained results are new and original. It should be mentioned that in addition to its importance from a fundamental standpoint, the present study finds important application in magma solidification, the melting of permafrost, the thawing of frozen grounds, and the preparation of semi-conductor materials. In addition, in solidifying casting the existence of a mushy zone separating the liquid phase from the solid phase has been observed.

Formulation of Problem. Consider a steady stagnation-point flow towards a horizontal linearly stretching sheet with the influence of magnetic field, melting at a steady rate into a constant property, warm liquid of the same material, as shown in the Figure above. It is assumed that the velocity of the external flow is \( u_e (x) = \alpha x \) and the velocity of the stretching sheet is \( u_a (x) = cx \), where \( \alpha \) is a positive constant, while \( c \) is a stretching rateis also a positive constant and \( x \) is the coordinate measured along the stretching sheet. It is also assumed that the temperature of melting surface is \( T_m \), while the temperature in the free-stream region is \( T_w \). Under the usual boundary layer approximations, the equations of motion with magnetic field effect, and thermal radiation, and the equation representing temperature distribution in the liquid flow must obey the following equations,

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 ,
\]

\[
u \left( \frac{\partial u}{\partial x} \right) + \nu \left( \frac{\partial u}{\partial y} \right) - u_e \frac{\partial u}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma R_e^2 u}{\rho} ,
\]

\[
u \left( \frac{\partial T}{\partial x} \right) + \nu \left( \frac{\partial T}{\partial y} \right) = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \left( \frac{\partial q_f}{\partial y} \right)
\]

where \( x \) and \( y \) – the Cartesian coordinates measured, along and normal to the stretching surface;
\( u \) and \( v \) – the velocity components along the \( x \)- and \( y \)-axis; 

\( \nu, \alpha, \) and \( q_r \) – the kinematic viscosity, thermal diffusivity and thermal radiation of the fluid, respectively.

Following Hiemenz [11] and Epstein and Cho [14], the boundary conditions of (1) – (3) are as mentioned below,

\[
\begin{align*}
\frac{d u}{dx} &= c x, & T &= T_m, & \text{at} & & y = 0 \\
\frac{d u}{dx} &= a x, & T &= T_c, & \text{as} & & y \to \infty
\end{align*}
\]

and

\[
\left( \frac{\partial T}{\partial y} \right)_{y=0} = \frac{\lambda}{\rho} \left( T_m - T_0 \right) v(x, 0)
\]

where \( \rho \) – fluid density; \( k \) is the thermal conductivity; 
\( \lambda \) – latent heat of the fluid 
\( c_s \) – heat capacity of the solid surface.

Eq. (4b) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid temperature \( T_0 \) to its melting temperature \( T_m \) (Refer, Epstein and Cho[14]). Following the classical work of Hiemenz [11], we introduce, a similarity transformation to recast the governing partial differential equations into a set of ordinary differential equations, i.e.

\[
\psi = \left( \frac{\nu}{c} \right)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_m}{T_c - T_m}, \quad \eta = \left( \frac{\nu}{c} \right)^{1/2} y,
\]

where \( \psi \) – the stream function, defined in the usual form as, \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \), which automatically satisfies the continuity equation (1).

By using this stream function \( \psi \), we obtain the values of velocity components,

\[
u = a x f'(\eta), \quad v = \left( \frac{\nu}{c} \right)^{1/2} f(\eta),
\]

Substituting (5) and (6) into (2) and (3) gives the following nonlinear ordinary differential equations:

\[
\begin{align*}
f'' + ff' - f'^2 - M_n f' + 1 &= 0, \\
\theta'' + \left( \frac{Pr}{1 + Np} \right) f \theta' &= 0,
\end{align*}
\]
where primes denote differentiation with respect to $\eta$ and $Pr$ is the Prandtl number and $M_w = \frac{\sigma B_0^2}{\rho \alpha}$ is the Magnetic parameter, and $N_r = \frac{16 \sigma^3 T^3}{3 K_0^2 K}$ is the Radiation parameter.

The boundary conditions (4) takes the form,

$$
\begin{align*}
 f'(0) &= \varepsilon, & \theta(0) &= 0, & Pr f'(0) + M \theta'(0) &= 0, \\
 f'(\infty) &= 1, & \theta(\infty) &= 1,
\end{align*}
$$

where $\varepsilon = \frac{c}{d}$ — velocity ratio ($\varepsilon > 0$) parameter

$M$ — dimensionless melting heat transfer parameter which is defined as

$$
M = \frac{c_p (T_c - T_m)}{\lambda c_s (T_m - T_0)}
$$

where $c_p$ — the heat capacity of the fluid at constant pressure.

The melting heat transfer parameter is a combination of the Stefan numbers $c_s (T_c - T_m) / \lambda$ and $c_s (T_m - T_0) / \lambda$ for the liquid and solid phases, respectively. It is worth mentioning that for $M=0$ (melting is absent), Eq. (7) reduces to the classical equation first derived by Hiemenz [11].

The physical quantities of interest are the skin friction coefficient $c_f$ and the local Nusselt number $Nu_w$ or melting rate (local heat flux) at the stretching surface defined as

$$
\begin{align*}
 c_f &= \frac{\tau_w}{\rho u_c}, & Nu_w &= \frac{q_w}{k (T_c - T_m)}
\end{align*}
$$

where $\tau_w$ and $q_w$ — the surface shear stress and the surface heat flux which are given by,

$$
\begin{align*}
 \tau_w &= \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, & q_w &= -k \left( \frac{\partial T}{\partial y} \right)_{y=0}
\end{align*}
$$

where $\mu$ — the dynamic viscosity of the fluid.

Using variables (5), we get

$$
Re_w^{V^2} c_f = f^\prime(0), \quad Re_w^{V^2} Nu_w = \theta'(0)
$$
where $Re_x = \frac{\alpha(x) \nu}{\nu}$ — the local Reynolds number.

Combining (4b) and (13), one can obtain the melting velocity at the stretching surface $v(x, 0)$, which is represented by the following equation, i.e.

$$v(x, 0) = -\frac{\alpha}{\nu} MNu_x$$  \hspace{1cm} (14)

This velocity is proportional to $x^{-1}$ and it shows that the melting process is faster near the stagnation-point of the stretching sheet.

**Numerical method.** Numerical solutions to the ordinary differential equation (7) and (8) with the boundary conditions (9) are obtained using the Runge-Kutta-Fehlberg method with shooting technique. The Numerical solution is obtained by setting different initial guesses for the values of skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$, where the velocity and temperature profiles satisfy the far field boundary conditions (9) asymptotically. These methods have been successfully used by the following authors to solve various problems related to boundary layer flow and heat transfer (see Bachok et al. [8, 9] and Ishak et al. [15]).

**Results and Discussion.** For the stretching sheet, $\nu$ is positive, in order to validate the numerical results obtained, we compare our results with that reported by Bachok et al. [16], Hiemenz [11] Wang [17] Kimiaefar et al. [18], as shown in Table 1, for $f''(0)$, which depicts an excellent agreement between our result and that of others as mentioned above which in turn gives a confidence that the numerical results obtained by us are accurate.

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**Effect of M on temperature and velocity profiles.** Influence of melting parameter $M$ on the temperature $\theta(\eta)$ is captured in fig. 1. It is obvious from this figure that an increase in the melting effect decreases the temperature. However, the thermal boundary layer thickness is increased for large values of $M$.

Figure 2 illustrates the influence of melting parameter $M$ on the velocity profile $f'\eta$. An increase in the melting parameter $M$ enhances the velocity and the boundary layer thickness. When a cold sheet plunges into a hot water it starts to melt. As the melting progresses, the sheet gradually transforms to a liquid causing the velocity profiles to grow rapidly.
Effect of Magnetic parameter $M_n$ on velocity and temperature profiles. Fig 3 shows the effect of Magnetic parameter $M_n$ on temperature profile. Clearly, increasing values of magnetic parameter $Mn$ causes the surface temperature to blow-up monotonically. Fig 4 shows the effect of magnetic parameter $M_n$ on dimensionless velocity profile $f'(\eta)$, and it is noticed that velocity profile of the fluid in the boundary layer significantly reduces with increase in values of $M_n$. This is because of the fact that, magnetic field introduces a retarding force which acts transverse to the direction of applied magnetic field. This force is Lorentz force, which decelerates the flow in the boundary layer, resulting in thickening momentum boundary layer and also it is noticed in increase of absolute value of velocity gradient at the surface of the sheet.

Effect of Radiation parameter on temperature and velocity profiles. The effect of radiation parameter $Nr$ on the velocity profile has been illustrated in Fig.5. It is noticed that radiation parameter $Nr$ is an increasing function of velocity profile $f'(\eta)$.

The effects of thermal radiation parameter $Nr$ on temperature is shown in Fig.6. It is revealed that the radiation parameter $Nr$ causes increase in the fluid temperature $\theta(\eta)$. On the other hand the thermal boundary layer thickness also increases.

Effect of prandtl number on temperature and velocity profiles. Fig.7-8 exhibits the effect of the Prandtl number on velocity and temperature profiles respectively. The temperature profile and thermal boundary layer thickness quickly decrease with increasing values of Pr. Prandtl number acts as a means to increase fluid viscosity resulting in lessening the flow velocity and temperature. Here thermal boundary layer thickness decreases with increasing Prandtl number, which is consistent with the findings of various researchers. Low prandtl number Pr indicates fluids with larger thermal conductivity and this produces thicker thermal boundary layer structures than that for high Prandtl number.

Summary It is clear that the boundary layer thickness increases as both the magnetic field effect and melting heat transfer increase. The velocity field $f'(\eta)$ is decreasing function of magnetic parameter $M_n$ where as it is increasing function of temperature profile. As expected, the effects of melting parameter $M$ and Pr on the velocity and temperature are opposite. The influence of stretching ratio $A$ is to increase the velocity and temperature fieldssignificantly. The present results are in a very good agreement with the numerical results obtained by Bachok et al. [16] for viscous fluid. Our analysis shows that the thermal radiation parameter groups with the Prandtl number, and the grouped dimensionless parameter has a complex impact on the heat transfer process.

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**Fig. 1.** Temperature $\theta(\eta)$ versus $\eta$ for different values of $M$  
**Fig. 2.** Velocity profile $f'(\eta)$ versus $\eta$ for different values of $M$
Fig. 3. Temperature profile $\theta(\eta)$ versus $\eta$ for different values of $M$

Fig. 4. Velocity profile $f'(\eta)$ versus $\eta$ with different values of $M$

Fig. 5. Velocity profile $f'(\eta)$ versus $\eta$ for different values of $Nr$

Fig. 6. Temperature profile $\theta(\eta)$ versus $\eta$ for different values of $Nr$

Fig. 7. Velocity profile $f'(\eta)$ versus $\eta$ for different values of Prandtl number $Pr$

Fig. 8. Temperature profile $\theta(\eta)$ versus $\eta$ for different values of $Pr$
References


