Analytical and Numerical Study of Foam-Filled Corrugated Core Sandwich Panels under Low Velocity Impact

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ABSTRACT. Analytical and finite element simulations are used to predict the effect of core density on the energy absorption of composite sandwich panels under low-velocity impact. The composite sandwich panel contains two facesheets and a foam-filled corrugated core. Analytical model is defined as a two degree-of-freedom system based on equivalent mass, spring, and dashpot to predict the local and global deformation response of a simply supported panel. The results signify a good agreement between analytical and numerical predictions.

Introduction. Sandwich panels have been widely used for constructing bridge decks, temporary landing mats and thermal insulation wall boards due to better performance in comparison to other structural materials in terms of enhanced stability, higher strength to weight ratios, better energy absorbing capacity and ease of manufacture and repair. In sandwich panels, low density material, known as core, is usually adopted in combination with high stiffness face sheets to resist high loads. The main functions of core materials are to absorb energy and provide resistance to face sheets to avoid local buckling [1]. For sandwich panels having corrugated cores, it has been envisioned that this may be achieved if proper lateral support to core members against plastic yielding and buckling is supplied. To this end, recently, Yan et al. [2] inserted high porosity close-celled aluminium foams into the interstices of corrugated sandwich panels made of 304 stainless steel. A combined experimental and numerical study of the hybrid-cored sandwich was carried out under quasi-static compressive loading. It was found that the foam filling into the core of an empty corrugated sandwich could increase the compressive strength and energy absorption capacity of the hybrid sandwich by as much as 211% and 300%, respectively, and the specific energy absorption by 157%.

Yan et al. [3] made theoretically and experimental studies on the behavior of sandwich beams with aluminum foam-filled corrugated cores under three-point bending. The bending stiffness, initial failure load and peak load of the sandwich structure were predicted by theoretical analysis. They concluded that the filling of aluminum foams led to dramatically increased bending stiffness, initial failure load, peak load, and sustained load-carrying capacity relative to an unfilled corrugated sandwich panel.

Yu et al. [4] investigated the crushing response and collapse modes of metallic corrugate-cored sandwich panels filled with close-celled aluminum foams using Finite Elements Method. They show that at low compression velocities, the foam-filled panel was more efficient in energy absorption...
compared to the empty panel due to the lateral support provided by the filling foam against strut buckling if the foam relative density was sufficiently large.

Yazici et al. [5] investigated experimentally the influence of foam infill on the blast resistivity of corrugated steel core sandwich panels and numerically through Finite Elements Method. After verifying the finite element model, numerical studies were conducted to investigate the effect of face sheet thickness, corrugated sheet thickness, and boundary conditions on the blast performance. Experimental and numerical results were found to be in good agreement with R2 values greater than 0.95. The greatest impact on blast performance came from the addition of foam infill, which reduced both the back-face and front-face deflections by more than 50% at 3 ms after blast loading at a weight expense of only 2.3%. Foam infill benefits were more prominent for Simple Supported edge case than Encastre Supported edge case.

Han et al. [6] explored the physical mechanisms underlying the beneficial effect of filling aluminum foams into the interstices of corrugated plates made of stainless steel with finite element simulations. Relative to unfilled corrugated plates of equal mass, this effect was assessed on the basis of elevated peak stress and enhanced energy absorption under quasi-static out-of-plane compression. Upon validating the FE predictions against existing measurements, the influence of key geometrical and material parameters on the compressive response of foam-filled corrugated plates was investigated. Four new buckling modes were identified for foam-filled corrugations. Based upon these deformation modes of post-buckling, collapse mechanism maps were constructed. Due to the additional resistance provided by foam filling against buckling of the corrugated plate and the strengthening of foam insertions due to complex stressing, both the load bearing capacity and energy absorption of foam-filled sandwiches were greatly enhanced.

In this paper, the effect of core geometry on the energy absorption of foam-filled corrugated core sandwich panels is investigated through analytical and numerical simulations.

1. Analytical study of composite sandwich panels

1.1. Static indentation

**Local deformation.** Rigidly supported sandwich panels experience only local deformation of top facesheet. Many of the analytical methods for determining the local deformation involve Hertzian contact methods [7]. Since the local deformation causes transverse deflections of the entire top facesheet and core crushing, that Hertzian contact laws are inappropriate for finding local indentation response. Other methods for determining local deformation and core compression include modeling the top facesheet on a deformable foundation [8,9]. Turk and Hoo Fatt [10] presented an analytical solution for the local indentation of a rigidly supported composite sandwich panel by a rigid, hemispherical nose cylinder. They modelled the sandwich composite as an orthotropic membrane resting on a rigid-plastic foundation model. The solution was found to be within 15% of experimental results that involved facesheet indentations that were several times the facesheet thickness [11].

In this paper, local indentation of a sandwich panel is found by considering the elastic, perfectly plastic core as a deformable foundation for the top facesheet. Fig. 1 shows three possible regimes of top facesheet indentation: (I) plate on an elastic foundation; (II) plate on a rigid-plastic foundation; (III) membrane in a rigid-plastic foundation. When the indentation is very small and core crushing is elastic the local indentation response is found by considering a plate on an elastic foundation. As the facesheet indentation becomes larger but still less than about half of the plate thickness, local indentation response is found using a plate on a plate on a rigid-plastic foundation. If the facesheet indentation is larger than the facesheet thickness, the local indentation response is found by considering a thin membrane on a rigid-plastic foundation.
Abrate [12] gives the following expression for the local indentation of a simply supported plate on the elastic foundation

\[
\delta_{\text{local}} = \sum_{m=1}^{n} \sum_{n=1}^{m} \left\{ \frac{4\rho \sin \left( \frac{m\pi}{2} \right) \sin \left( \frac{n\pi}{2} \right)}{\rho} \right\} \left\{ \frac{\pi}{\rho} (J_{1m} n^2 + 2(J_{12} + 2J_{22}) n^2 + k^2) \right\}
\]  

(1)

where \(J_{ij}\) is the bending stiffness of the laminate face-sheet;

\[k^2 = E_{\text{core}} / H\]  

is the transverse elastic stiffness of the core.

**Plate on rigid-plastic foundation.** Fatt and Park [13] obtained the load-indentation by using the principle of minimum potential energy. The total potential energy \(\Pi\) is given by

\[
\Pi = U + D - V
\]  

(2)

where \(U\) is the strain energy due to bending;

\(D\) is the work due to core crushing;

\(V\) is the work done by the indentation force.

Assume that the local indentation is only due to bending and has the form

\[
w(x, y) = \begin{cases} 
\delta \text{ for } 0 \leq x^2 + y^2 \leq R^2 \\
\delta \left[ 1 - \left( \frac{x}{\zeta - R} \right)^2 \right] \left[ 1 - \left( \frac{y}{\zeta - R} \right)^2 \right] \\
\delta R^2 \leq x^2 + y^2 \leq \zeta^2, x \geq 0, y \geq 0
\end{cases}
\]  

(3)

where \(\delta\) is the deflection under the indenter;

\(\zeta\) is the lateral extent of deformation;
$R$ – is the radius of the indenter.

The above function is defined only in the positive quadrant. It is assumed that the profile is symmetric with respect to both x- and y-axis. Coefficients of the above polynomial function were chosen to satisfy the boundary conditions as follows:

Zero slope surrounding the projectile nose:

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0 \quad \text{at} \quad 0 \leq x^2 + y^2 \leq R^2$$

Zero slope and deflection at the boundary of the deflection zone:

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = 0, \quad w = 0 \quad \text{at} \quad x^2 + y^2 = \zeta^2$$

The strain energy due to bending of an orthotropic laminate facesheet is:

$$U \approx \frac{1}{2} \int \left[ D_{11} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_{12} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + 4D_{66} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dA$$

where $D$ – is the laminate bending stiffness matrix,

$dA = dxdy$ and $A$ – is the surface area of the deformed facesheet.

The integral can be approximated as

$$U \approx 2 \int \int \left[ D_{11} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + D_{22} \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_{12} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + 4D_{66} \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dxdy$$

Substituting derivatives of Eq. (3) into Eq. (5) gives

$$U = \frac{D_1 \delta^2}{(R - \zeta)^2}$$

where

$$D_1 = \frac{16384}{11025} (7D_{11} + 7D_{22} + 8D_{66})$$

$D_1$ is the bending stiffness of the orthotropic facesheet. The work due to core crushing is also approximated by
where \( q \) is the crushing strength of the core.

Using the assumed profile in Eq. (3), one gets

\[
D = \pi q R^2 \delta + \frac{256}{255} q \delta (R - \zeta)^2
\]  

(9)

The work done by the indentation force is

\[
V = P \delta
\]  

(10)

Therefore, the total potential energy is

\[
\Pi = \frac{D q \delta^2}{(R - \zeta)^2} + \pi q R^2 + \frac{256}{255} q (R - \zeta)^2 - P \delta
\]  

(11)

Minimizing \( \Pi \) with respect to \( \delta \) gives

\[
P = \frac{2Dq \delta}{(R - \zeta)^2} + \pi q R^2 + \frac{256}{255} q (R - \zeta)^2
\]  

(12)

Likewise minimizing \( P \) with respect to \( \zeta \) gives

\[
\frac{2Dq \delta}{(R - \zeta)^2} = \frac{256}{255} q (R - \zeta)^2
\]  

(13)

Eliminating the length of the deformation zone from Eqs. (11) and (12) gives the load-indentation response as

\[
P = 32 \sqrt\frac{2}{255} D q \delta + \pi q R^2
\]  

(14)

The first term is the resistance due to facesheet bending and crushing of core outside the contact area of indenter, while the second term is due to crushing of core under the indenter.
Membrane on rigid-plastic foundation. The force-deformation relation is obtained as

\[ P = \frac{8}{3} \sqrt{C_1 q} \delta^2 + \pi q R^2 \]  

(15)

Where

\[ C_1 = 8 \left[ \frac{(A_{11} + A_{22})}{45} + \frac{(2A_{12} + 4A_{66})}{49} \right] \]

The first term in Eq. (15) is the resistance due to membrane stretching and crushing of honeycomb outside the contact surface if indenter, while the second term is due to crushing of honeycomb under the indenter. Also, a relation between the local indentation and the extent of deformation is given by:

\[ \delta = \left[ \frac{q(\zeta - R^d)}{9C_1} \right]^{\frac{1}{2}} \]  

(16)

Global deformation. When the panel is clamped around the edges, it experiences the two types of deformations: (1) local deformation of the top facesheet into the core material, \( \delta \), and (2) global panel bending and shear deformation, \( \Delta \). The local deformation is the local indentation of the top facesheet as the core crushes. The global deformation is understood as the bending and shear deformation of a sandwich panel that has not experienced any local facesheet indentation and core crushing. In reality, both the local and global deformations are coupled.

The principle of minimum potential energy is again to derive approximate solutions for simply supported panels. Functions describing the transverse deformation, \( W \) and the rotations, \( \alpha \) and \( \beta \), are approximated from the exact series solution of a simply supported composite sandwich panel subjected to a point load at its center. Using the actual series solution for the deformations is not practical because a very large number of terms would have to be retained before convergence of the series solution. The resulting trial functions are as follows [14]:

\[ w(x, y) = \Delta \left[ 1 - \left( \frac{2y}{a} \right)^2 \right] \left[ 1 - \left( \frac{2y}{a} \right)^2 \right] \]

for \(-a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2\)  

(17)

\[ \alpha(x, y) = \alpha_1 \left[ -\frac{3x}{a} + 4 \left( \frac{y}{a} \right)^3 \right] \left[ 1 - \left( \frac{2y}{a} \right)^2 \right] \]

for \(-a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2\)  

(18)

\[ \beta(x, y) = \beta_1 \left[ -\frac{3y}{a} + 4 \left( \frac{x}{a} \right)^3 \right] \left[ 1 - \left( \frac{2x}{a} \right)^2 \right] \]

for \(-a/2 \leq x \leq a/2, -a/2 \leq y \leq a/2\)  

(19)
Note that these trial functions satisfy the boundary conditions, i.e., \( w = 0, \frac{d\alpha}{dx} = 0, \frac{d\beta}{dy} = 0 \) at the edges. In-plane deformations are assumed to be negligible with respect to the transverse deformation. Thus, the strain energy for a symmetric sandwich panel is given by:

\[
U = 4 \int_0^l \left\{ \frac{1}{2} \left( \frac{\partial \alpha}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial \beta}{\partial y} \right)^2 + \frac{A_1}{2} \left( \frac{\partial \alpha}{\partial x} \right)^2 + A_3 \left( \frac{\partial \alpha}{\partial x} \right)^2 + \frac{1}{2} (\frac{\partial w}{\partial x})^2 \right\} dx dy
\]

(20)

Substituting derivatives of the expressions in Eqs. (17)-(19) into Eq. (20) gives the following expression for the strain energy:

\[
U = F_1 A^2 + F_2 A \beta_0 + F_3 \beta_0^2 + F_4 \Delta \alpha_0 + F_5 \alpha_0^2 + F_6 \alpha_0 \beta_0
\]

(21)

where

\[
F_1 = \frac{2240}{1575} (A_{44}^2 + A_{55}^2)
\]

\[
F_2 = \frac{1344}{1575} \alpha A_{44}^2
\]

\[
F_3 = \frac{1}{1575} (204 \alpha^2 A_{44}^2 + 2016 D_{11}^\alpha + 2040 D_{12}^\alpha)
\]

\[
F_4 = \frac{1344}{1575} \alpha A_{55}^2
\]

\[
F_5 = \frac{1}{1575} (204 \alpha^2 A_{55}^2 + 2016 D_{11}^\alpha + 2040 D_{12}^\alpha)
\]

\[
F_6 = \frac{4032}{1575} (D_{11}^\alpha + D_{12}^\alpha)
\]

Thus we have

\[
\Pi = F_1 A^2 + F_2 A \beta_0 + F_3 \beta_0^2 + F_4 \Delta \alpha_0 + F_5 \alpha_0^2 + F_6 \alpha_0 \beta_0 - P \Delta
\]

(22)

Minimizing \( \Pi \) with respect to \( A \) and \( \beta_0 \) ensures equilibrium of the system and yields the load-indentation response. Minimizing \( \Pi \) with respect to \( \Delta \) gives

\[
\frac{\partial \Pi}{\partial \Delta} = 2F_1 \Delta + F_2 \beta_0 + F_4 \alpha_0 - P = 0
\]

(23)

Likewise minimizing \( \Pi \) with respect to \( \alpha_0 \) gives
Minimizing $\mathcal{I}$ with respect to $\beta_0$ gives

$$\frac{\partial \mathcal{I}}{\partial \beta_0} = F_2 \Delta + 2 F_3 \beta_0 + F_4 \alpha_0 = 0$$

(25)

The global load-deflection response is found by eliminating $\alpha_0$ and $\beta_0$ in Eqs. (24), (25). Hence:

$$P = K_g \Delta$$

(26)

where

$$K_g = \frac{K_{sd} \Delta^2}{(A F_1 F_2 - F_2^2)(A F_1 F_3 - F_3^2) + (2 F_2 F_3 - F_2 F_4)} + \frac{F_2 F_3 - 2 F_2 F_4}{[2 F_2 (A F_1 F_3 - F_3^2)]}$$

1.2. Low-velocity impact on simply supported sandwich panels. The following section described simple dynamic models for the impact response of simply supported sandwich panels. Regarding to Fig. 2 the equations of motion for the 2-DOF system are

$$(M_0 + m_f) \ddot{\delta} + P_1(\delta) + Q_d = 0$$

(27)

and

$$P_1(\delta) + Q_d = m_s \ddot{\Delta} + K_g \Delta$$

(28)

where $Q_d$ is the dynamic crushing resistance of the core that can be experimentally evaluated. $m_t$ is the effective mass of the top facesheet, and the effective mass of the sandwich is $m_s \cdot K_g$. $K_g$ is the dynamic global stiffness of the sandwich.
The above equations would be difficult to solve because of the nonlinear local spring response. Assume that local spring response can be linearized

\[ p_i(\delta) \approx K_{i\delta} \delta \]  

where \( K_{i\delta} \) is the dynamic local stiffness of top facesheet.

Also assume again the mass of sandwich panel is negligible compared to the mass of the projectile for simplicity. Therefore Eqs. (27) and (28) simplify to

\[ M_\delta(\ddot{\delta} + \delta) + K_{\delta\gamma} \delta + Q_\delta = 0 \]  

And

\[ K_{i\delta} \dot{\delta} + Q_i = K_{i\delta} \Delta \]  

Differentiating both sides of Eq. (31) twice with respect to time gives

\[ \ddot{\Delta} = \frac{K_{i\delta}}{K_{\delta\gamma}} \ddot{\delta} \]  

Substituting \( \ddot{\Delta} \) into Eq. (30) gives

\[ M_\delta(1 + \frac{K_{i\delta}}{K_{\delta\gamma}}) \ddot{\delta} + K_{\delta\gamma} \delta + Q_\delta = 0 \]  

Also by differentiating both sides of Eq. (31) with respect to time and setting \( f = 0 \) we obtain
\[
\dot{\delta}_0 = \frac{K_{\text{sd}}}{K_{\text{jd}}} \dot{\delta}_0
\]  

(34)

According to the momentum conservation law, \( M_o \dot{V}_0 = M_o \dot{\ddot{X}_1} \) where \( \dot{\ddot{X}_1} \) is the velocity of the upper facesheet obtained from \( \dot{\dot{X}_1} = \dot{\delta}_0 + \dot{\dot{\delta}}_0 \). Thus the initial condition is \( \dot{\delta}_0 = \dot{\dot{\delta}}(0) = \frac{K_{\text{sd}}V_0}{K_{\text{vd}} + K_{\text{jd}}} \) and \( \dot{\delta}(0) = 0 \).

The solution for \( \delta \) is given by:

\[
\delta = \frac{\dot{\delta}}{\omega} \sin \omega t + \frac{Q_j}{K_{\text{jd}}} \cos \omega t - \frac{Q_j}{K_{\text{jd}}}
\]

(35)

Where

\[
\omega = \sqrt{\frac{K_{\text{vd}}K_{\text{jd}}}{(K_{\text{jd}} + K_{\text{sd}})M_0}}.
\]

The velocity and acceleration of top facesheet is found by differentiating Eq. (35).

The impact force is given by

\[
F(t) = -M_o(\ddot{\delta} + \dot{\delta}) = -M_o(1 + \frac{K_{\text{vd}}}{K_{\text{sd}}})\dot{\delta}
\]

(36)

The maximum impact force occurs when \( \frac{dF}{dt} = 0 \) and is given by

\[
F_{\text{max}} = \frac{M_o}{K_{\text{sd}}} \left( \frac{(K_{\text{vd}} + K_{\text{sd})})\omega}{\sqrt{(Q_j\omega)^2 + \left(\dot{\delta}^2 K_{\text{jd}} + \frac{Q_j^2 \omega^2}{K_{\text{jd}}}} \right)}
\]

(37)

Maximum impact force occurs when

\[
I_{\text{min}} = \frac{1}{\omega} \tan^{-1} \left( \frac{\dot{\delta}_0 K_{\text{jd}}}{Q_j\omega} \right)
\]

(38)

Maximum strain rate is also given by
\[ \dot{e}_{\text{max}} = \frac{e_{\text{max}}(\theta)}{\tan^{-1} \left( \frac{S_{c} K_{c}}{Q_{0}, \omega} \right)} \]  

(39)

2. Analytical study of sandwich panels with corrugated sandwich panels. Fig. 3 shows a sandwich panel with corrugated core.

![Fig. 3. Corrugated lattice sandwich structure unit cell dimensions.](image)

The core density of triangular sandwich structure are formulated respectively as [15]

\[ \rho_{c} = \frac{2t_{1}}{L\sin 2\theta} \rho \]  

(40)

where \( \rho \) – is the density of the base material of the core sheets, \( L = H / \sin \theta \) for triangular core as in Fig. 4. Thus, the relative density for the triangular core can be expressed as [16]

\[ \bar{\rho} = \frac{2t}{t \sin 2\theta} \]  

(41)

![Fig. 4. Geometry of triangular core.](image)

For a foam-filled corrugated core, the total average density of the sandwich core may be expressed as [2]:

\[ \rho_{\text{total}} = \rho_{c} \bar{\rho}_{c} + \rho_{f}(1 - \bar{\rho}_{c}) \]  

(42)
where \( V_c \) – is the volume proportion of the core occupied by corrugated plate;
\( \rho_f \) – is the density of foam.

Then the total average density of the sandwich core can be written as

\[
\rho_{\text{core}} = \frac{2t_i}{l \sin 2\theta} V_c + \rho_f (1 - V_c)
\]  

The overall shear deflection of web-foam core is the sum of the web and foam shear deflections. Based on the static relationship [17]:

\[
\tau_w = \tau_w V_w + \tau_f V_f
\]  

where \( \tau_w \), \( \tau_w \) and \( \tau_f \) – are the shearing stress of web-foam core, web and foam, respectively;
\( V_w \) and \( V_f \) – the volume ratio of web and foam, respectively.

The geometrical relationship:

\[
\gamma_{xy} = \gamma_w = \gamma_f
\]  

where \( \gamma_{xy} \), \( \gamma_w \) and \( \gamma_f \) – the shear strain of web-foam core, web and foam, respectively.

Using Hooke’s law, the corresponding stresses are

\[
\tau_{xy} = \gamma_{xy} G_{xy}
\]
\[
\tau_w = \gamma_w G_w
\]
\[
\tau_f = \gamma_f G_f
\]

where \( G_{xy} \), \( G_w \) and \( G_f \) – the shear modulus of web-foam core, web and foam, respectively.

The elastic modulus of the corrugation when loaded in \( x_3 \) direction can be expressed as [18]:

\[
E_{\chi} = E_i \bar{\rho} \sin^4 \theta
\]

where \( E_i \) – the Young’s modulus of the parent material.

Using the same method, the effective shear modulus of the corrugated core, \( G_i \) can be expressed as
For a foam-filled corrugated core the elastic modulus is given by

\[ E_{\text{total}} = E_c \nu_c + E_f (1 - \nu_c) = \frac{2t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + E_f (1 - \nu_c) \]  

(49)

Compressive strength, \( \sigma_3 \), as well as transverse shear strength, \( \sigma_4 \), of the corrugated core

\[ \sigma_3 = \sigma_c \bar{\rho} \sin^2 \theta = \sigma_c \sin^2 \theta \left( \frac{2t_c \nu_c}{L \sin 2\theta} + \rho_f (1 - \nu_c) \right) \]  

(50)

\[ \sigma_4 = \frac{\sigma_c \rho}{2} \sin 2\omega = \frac{\sigma_c}{2} \sin 2\omega \left( \frac{2t_c \nu_c}{L \sin 2\theta} + \rho_f (1 - \nu_c) \right) \]  

(51)

The dynamic global stiffness \( K_{\text{gd}} \) for a simply supported sandwich panel is given by [19]:

\[ K_{\text{gd}} = \frac{(4F_3 F_5 - F_4^2) (4F_4 F_5 - F_3^2) + (2F_2 F_3 F_4 - F_2 F_5)}{(F_2 F_6 - 2F_2 F_5) / [2F_2 (4F_3 F_5 - F_4^2)]} \]  

(52)

Where

\[ F_1 = \frac{2240}{1575} \frac{4t_c E_c \sin^4 \theta}{L \sin 2\theta} \nu_c + 2E_f (1 - \nu_c) \]  

\[ F_2 = \frac{1344}{1575} \frac{2t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + E_f (1 - \nu_c) \]  

\[ F_3 = \frac{1}{1575} \left( 204 \left[ \frac{2t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + E_f (1 - \nu_c) \right] + \frac{4t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + 2E_f (1 - \nu_c) \right) \]  

\[ F_4 = \frac{1344}{1575} \frac{2t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + E_f (1 - \nu_c) \]  

\[ F_5 = \frac{1}{1575} \left( 204 \left[ \frac{2t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + E_f (1 - \nu_c) \right] + \frac{4t_c E_c \sin^2 \theta}{L \sin 2\theta} \nu_c + 2E_f (1 - \nu_c) \right) \]  

\[ F_6 = \frac{4032}{1575} \frac{4t_c E_c \sin^4 \theta}{L \sin 2\theta} \nu_c + 2E_f (1 - \nu_c) \]
Thus we obtain the final stiffness

\[ K_{g\_new} = \left[ (4F_{1}F_{3} - F_{2}^{2})(4F_{5}F_{6} - F_{4}^{2}) + (2F_{1}F_{5} - F_{4}F_{6}) \right] \]

\[ (F_{5}F_{6} - 2F_{2}F_{4}) / [2(4F_{1}F_{5} - F_{4}^{2})] \]

As well as the deformation, velocity and acceleration in terms of time:

\[ \delta_{n+1} = \frac{\delta_n \sin \omega t + \frac{Q}{K_{1,d}} \cos \omega t - \frac{Q}{K_{\infty}}}{K_{1,d} \omega} \]

3. Numerical study

**Numerical modelling of corrugated-core sandwich panels.** The corrugated sandwich panels were analysed using the explicit FE code ANSYS/LS-DYNA. The face sheets and sandwich cores were made of Al-1000 aluminum alloy. The corrugated core members were meshed by structural shell element S4R and quadratic structural element. The detailed material parameters are summarized in Tables 2 and 3.

With symmetry boundary conditions, displacement controlled quasi-static uniaxial compression was applied to the top face sheet while the bottom face sheet was fixed.

Upon performing a mesh sensitivity study, an element size on the order of 1.5 was shown to be sufficiently refined for ensuring the accuracy of the numerical results. The upper indenter was simulated by using eight-node solid elements, and the lower platform was defined to be rigid. An automatic surface-to-surface contact was defined between the upper indenter and the sandwich panel. Meanwhile, an automatic single surface contact was considered to simulate self-contact of core sheets during deformation. An automatic one-way surface-to-surface contact was defined between the face sheets and core members. For this reason, a speed of 2 m/s was adopted in the simulation. FE model of the triangular corrugated sandwich panel is shown in Fig. 5.

**Fig. 5. Deformation and stress distribution in Finite element model of triangular corrugated sandwich panel.**

**Numerical modeling of sandwich panels with foam core.** This section is intended to give a brief review on the capabilities of LS-DYNA finite element code for simulation of impact event. The numerical simulation is used for interaction between a rigid impactor and a sandwich structure with aluminum foam-core during impact. Impactor is modeled and meshed using quad elements as shown
in Fig. 7. The impactor is modeled using the material type 20 (rigid). Fig. 2 shows the model of steel impactor in LS-DYNA. Material constants for the steel impactor are presented in Table 1.

Table 1. Properties of steel impactor.

<table>
<thead>
<tr>
<th>Material property</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>E (GPa)</th>
<th>$\nu$</th>
<th>$\sigma_Y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>7800</td>
<td>210</td>
<td>0.3</td>
<td>400</td>
</tr>
</tbody>
</table>

Fig. 6. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for a triangular corrugated-core sandwich panel.
Plastic-kinematic model with material number 3 is used for Aluminum plate while Aluminum foam is modelled using the Deshpande-Fleck foam model by choosing material number 154 in LS-DYNA [20, 21, 22]. Fig. 8 shows the model of Aluminum plate in LS-DYNA. Material constants for the Aluminum are presented in Table 2.

Fig. 9 shows the model of Aluminum foam in LS-DYNA. Material constants for the Aluminum foam are presented in Table 3.

**Table 2. Properties of Aluminum.**

<table>
<thead>
<tr>
<th>Material property</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>E(GPa)</th>
<th>$\nu$</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>2700</td>
<td>70</td>
<td>0.3</td>
<td>117</td>
<td>124</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Fig. 9. A view of Aluminum foam model in LS-DYNA.

Table 3. Properties of Aluminum foam.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Relative Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(18%)</td>
</tr>
<tr>
<td>E (MPa)</td>
<td>1500</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.1</td>
</tr>
<tr>
<td>$\gamma$ (MPa)</td>
<td>4.3</td>
</tr>
<tr>
<td>$\varepsilon_{\text{f}}$</td>
<td>1.63</td>
</tr>
<tr>
<td>$\alpha_2$ (MPa)</td>
<td>48</td>
</tr>
<tr>
<td>$B$</td>
<td>5.5</td>
</tr>
<tr>
<td>$\sigma_{\text{f}}$ (MPa)</td>
<td>3.8</td>
</tr>
<tr>
<td>$\varepsilon_{\text{cr}}$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

In some models such as Deshpande-Fleck foam model it may be not possible to reduce the step time. In order to solve this problem in LS-DYNA the element erosion method is used to remove the heavily distorted elements. Several criteria are used to this end. Although in the present work the maximum strain criterion is utilized, the maximum stress criterion is applicable. For the case of Aluminum foam the maximum strain of 0.3 is used from the experimental results [23]. "MAT-add-erosion" is an auxiliary tool to remove the elements of impressed region [24,25,26].
Fig. 10. Deformation and stress distribution in finite element model of foam-core sandwich panel.

Fig. 11. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for foam-core relative density of 18%.
Fig. 12. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for foam-core relative density of 23%.
Fig. 13. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for foam-core relative density of 27%.

**Numerical modelling of sandwich panels with corrugated foam-filled core.** In the case of the foam-filled panel, symmetry boundary condition was applied on the two side faces of the foam insertion. Both the front and back face sheets of the sandwich were assumed to be stiff enough to be modelled as rigid bodies. Both the corrugated core members and the filled foam were meshed by structural shell element S4R. The foam insertions, the face sheets as well as the struts were also perfectly bonded at the interface [27].
Fig. 14. Deformation and stress distribution in finite element model of sandwich panel with corrugated foam-filled core.

Fig. 15. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for a sandwich panel with corrugated foam-filled core with relative density of 18%.
Fig. 16. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for a sandwich panel with corrugated foam-filled core with relative density of 23%
Fig. 17. Time-variations of impactor displacement, velocity, and acceleration, imposed force of impactor, and impactor kinetic energy for a sandwich panel with corrugated foam-filled core with relative density of 27%.

**Summary.** Analytical and numerical methods were used to characterize the failure response of foam-filled corrugated core sandwich panels under low velocity impact. A two degree-of-freedom is used to analytically predict the local and global deformation behaviour of a simply supported panel. The effect of foam-core relative density on the impact properties of sandwich panels was studied. It was shown that the impact resistance and rate of energy absorption would be increased by densifying the foam-core. Also the results revealed a good correlation between the analytical and numerical predictions.

**Nomenclature**
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>length of panel</td>
</tr>
<tr>
<td>$A$</td>
<td>panel surface area</td>
</tr>
<tr>
<td>$A_{ij}$</td>
<td>laminate extensional stiffness matrix</td>
</tr>
<tr>
<td>$A_{ij}^d$</td>
<td>laminate dynamic extensional stiffness matrix</td>
</tr>
<tr>
<td>$A_{ij}^s$, $A_{ij}^s$</td>
<td>transverse shear stiffness of sandwich</td>
</tr>
<tr>
<td>$b$</td>
<td>width of panel</td>
</tr>
<tr>
<td>$C_1$</td>
<td>static membrane stiffness of laminate</td>
</tr>
<tr>
<td>$C_{1d}$</td>
<td>dynamic membrane stiffness of laminate</td>
</tr>
<tr>
<td>$D$</td>
<td>work in crushing core</td>
</tr>
<tr>
<td>$D_1$</td>
<td>static bending stiffness of laminate</td>
</tr>
<tr>
<td>$D_{1d}$</td>
<td>dynamic bending stiffness of facesheet</td>
</tr>
<tr>
<td>$D_{ij}$</td>
<td>laminate bending stiffness matrix</td>
</tr>
<tr>
<td>$D_{ij}^d$</td>
<td>dynamic bending stiffness matrix</td>
</tr>
<tr>
<td>$D_{ij}^s$</td>
<td>sandwich bending stiffness matrix</td>
</tr>
<tr>
<td>$E$</td>
<td>bending stiffness of the sandwich beam</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>$F_{max}$</td>
<td>Young's modulus of the parent material</td>
</tr>
<tr>
<td>$h$</td>
<td>maximum impact force</td>
</tr>
<tr>
<td>$H$</td>
<td>facesheet thickness</td>
</tr>
<tr>
<td>$k_c$</td>
<td>core thickness</td>
</tr>
<tr>
<td>$K_c$</td>
<td>transverse stiffness of core</td>
</tr>
<tr>
<td>$K_{gl}$</td>
<td>global stiffness of clamped panel</td>
</tr>
<tr>
<td>$K_{1d}$</td>
<td>dynamic global stiffness of clamped panel</td>
</tr>
<tr>
<td>$K_E$</td>
<td>dynamic local stiffness of top facesheet</td>
</tr>
<tr>
<td>$m_1$</td>
<td>kinetic energy</td>
</tr>
<tr>
<td>$m_s$</td>
<td>effective mass of top facesheet</td>
</tr>
<tr>
<td>$M_o$</td>
<td>effective mass of sandwich</td>
</tr>
<tr>
<td>$P$</td>
<td>projectile mass</td>
</tr>
<tr>
<td>$q$</td>
<td>indentation force</td>
</tr>
<tr>
<td>$q_d$</td>
<td>equivalent nonlinear spring response for top facesheet deformation</td>
</tr>
<tr>
<td>$Q_d$</td>
<td>static crushing strength</td>
</tr>
<tr>
<td>$Q_{ij}$</td>
<td>dynamic crushing strength</td>
</tr>
<tr>
<td>$R$</td>
<td>laminate or core stiffness matrix</td>
</tr>
<tr>
<td>$S$</td>
<td>blunt projectile radius</td>
</tr>
<tr>
<td>$t$</td>
<td>shear stiffness of the core</td>
</tr>
<tr>
<td>$t_{max}$</td>
<td>time</td>
</tr>
<tr>
<td>$U$</td>
<td>ply thickness</td>
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<tr>
<td>$V$</td>
<td>time for maximum deflection</td>
</tr>
<tr>
<td>$V_0$</td>
<td>total strain energy</td>
</tr>
<tr>
<td>$W$</td>
<td>work done by external forces</td>
</tr>
<tr>
<td>$w$</td>
<td>projectile velocity</td>
</tr>
<tr>
<td>$w_{top}$</td>
<td>local (top facesheet) indentation</td>
</tr>
<tr>
<td>$x, y$</td>
<td>top facesheet deflection</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>global (panel) deflection</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>in-plane coordinates of sandwich panel</td>
</tr>
<tr>
<td>$\beta$</td>
<td>shear angle along x-axis</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>amplitude of shear angle along x-axis</td>
</tr>
<tr>
<td>$\delta$</td>
<td>shear angle along y-axis</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>amplitude of shear angle along y-axis</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>amplitude of top facesheet velocity</td>
</tr>
<tr>
<td>$\dot{x}$</td>
<td>initial velocity of top facesheet</td>
</tr>
<tr>
<td>$\dot{\varepsilon}$</td>
<td>amplitude of global panel deformation</td>
</tr>
<tr>
<td>$\dot{\varepsilon}$</td>
<td>amplitude of overall panel velocity</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>initial velocity of panel</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>impact duration</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>strain</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>critical strain</td>
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<tr>
<td>$\dot{\varepsilon}$</td>
<td>densification ratio</td>
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<tr>
<td>$\dot{\varepsilon}$</td>
<td>strain rate</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Poisson's ratios of sandwich beam</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>total potential energy</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of facesheet</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of sandwich</td>
</tr>
<tr>
<td>$\rho_{total}$</td>
<td>total average density of the sandwich core</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>plateau stress</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>yield stress</td>
</tr>
<tr>
<td>$\omega$</td>
<td>frequency of vibration due to impact</td>
</tr>
<tr>
<td>$\psi$</td>
<td>extent of local indentation</td>
</tr>
<tr>
<td>$\dot{\psi}$</td>
<td>time derivative</td>
</tr>
</tbody>
</table>
References


Cite the paper