Mechanics, Materials Science & Engineering, Vol. 20 2019 – ISSN 2412-5954

SENSITIVITY ANALYSIS FOR THE MODEL OF HEAT EXPLOSION FOR THERMALLY INSULATED AND CONDUCTING SYSTEMS

Irina Viktorova[1], Sofya Alekseeva[1], Michael Wolfersberger[2], Aaron Roach[2], Lilith Nick[3] ✉

1 – School of Mathematical and Statistical Sciences, Clemson University, Clemson SC, USA

2 – Department of Mechanical Engineering, Clemson University, Clemson SC, USA

3 – Glenn Department of Civil Engineering, Clemson University, Clemson SC, USA

lnick@g.clemson.edu

DOI **10.2412/mmse.38.79.483** provided by Seo4U.link

Keywords: heat explosion, sensitivity analysis, Fourier's Law, mechanical failure, thermal failure, cyclic loading.

ABSTRACT. The mechanical behavior of materials is temperature dependent. The unique thermal failure phenomenon known as heat explosion is a result of a specific combination of material properties and temperatures. The goal for this paper is to revisit a specific previously created model, and improve it by performing a sensitivity analysis on the input parameters to determine which parameter affects the occurrence of heat explosion the most. This is done by computational analysis of thermoviscoelastic parameters to determine how the critical values at which heat explosion occurs are affected by different inputs. Then, that data was used to perform a sensitivity analysis through the use of partial derivatives and vector field to determine which parameter had the highest degree of effect on the critical values. The results of this research make modeling heat explosion more realistic, and designing for it easier.

Introduction. Heat explosion is the catastrophic failure of a material due to a buildup of heat inside. According to the two primary laws of heat conduction – Fourier's Law and Maxwell's Law, heat will dissipate proportionally from areas of high temperatures to areas of low temperatures. These laws also state that heat will disperse at the same rate it is generated. This is mostly true, but under certain conditions the opposite can be seen. In these cases, heat builds up in the material because it is being generated faster than it is being dispersed. This build up leads to heat explosion. Heat explosion is important to consider when designing mechanical systems, as it can cause a material to fail in a less obvious way than normal mechanical failures [1].

Heat explosion is an important design constraint in many engineering applications. This can range from the transportation industry to the medical industry. In these applications, it is necessary to predict the failure caused by heat explosion. Typically, this would be done by making assumptions about the material and environment, but these assumptions decrease the accuracy of the predictions. This paper develops a sensitivity analysis of the parameters of the model discussed in "Analysis of Heat Explosion for Thermally Insulated and Conducting Systems" [2].

The model derives an approach to predict thermal failure while limiting the amount of assumptions made. The parameters used are known as thermoviscoelastic properties, or viscoelastic properties that account for the influence of temperature. There are several parameters used in this model. The parameter for the heat sensitivity of the material is γ. Another description of this parameter is the heat retained by the system. The property for heat dissipated by the system is β. This parameter is dependent on the insulation of the material. A combination of these two parameters creates δ, which is the influence of heat on the material. δ^* is known as delta critical, and is the δ at the instant prior to heat explosion of the material [3]. T is the temperature of the system and T_m is the temperature of the surrounding media. The ratio of these temperatures used in integration is η. These parameters are currently unit-less and will be given more definite meaning in future work.

Sensitivity analysis is a technique used to determine how a set of variables affects an output. This paper shows the sensitivity analysis performed on δ*, γ, and β. The purposes of this sensitivity analysis was to determine which material property is the most important factor in heat explosion. The method of partial differentiation was used to determine which material property affected it most greatly.

Equations

Equation (1) is the Fourier modeling equation for heat transfer developed by Viktorova [4].

$$\delta^* = \left\{\frac{1+\gamma}{2}\left[T_m^{\frac{1-\gamma}{2}} \int_{\frac{1}{T_m}}^{1} \frac{d\eta}{\sqrt{1-\eta^{1+\gamma}}}\right]\right\} \tag{1}$$

This equation was used to find the delta critical value, or the heat influence of a material that causes heat explosion. In this equation, heat removal is assumed to be zero.

Equation (2) models this same idea, but in scenarios where β is no longer zero.

$$\delta^* = \left\{\frac{1+\gamma}{2}\left[\int_{1}^{T_m} \frac{dT}{\sqrt{\left[T_m^{1+\gamma} - T^{1+\gamma}\right] + \left[\frac{\beta(1+\gamma)}{2}(T-T_m)(T+T_m-2)\right]}}\right]\right\} \tag{2}$$

This version of the modeling equation is more applicable in situations. This is because in most situations, materials have some sort of cooling method in place to remove heat and therefore prevent or prolong overheating. In these situations, the heat removal parameter cannot be neglected or the resulting predictions will be inaccurate.

Equation (3) and Equation (4) are partial derivative definitions.

$$f_x(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h} \tag{3}$$

$$f_x(x, y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h} \tag{4}$$

These partial derivative equations were used in the sensitivity analysis to create a vector field, which revealed which parameters affect the occurrence of heat explosion the most.

Results and Discussion

Delta Critical

Figure 1 shows delta values based on T_m for Equation (1). The beta values were set to zero based on the model, and ten different values of gamma were tested. The graphs show that the delta values rise until reaching delta critical, and then decrease as T_m increases. It also shows that as the gamma values increase, the peaks happen at smaller delta and T_m values. This is because gamma represents the heat retained by the material. With this increase in heat retained, heat explosion will occur at a lower temperature. These subplots differ from the plots produced in the previous analysis, as the model was found to be undefined at values less than one.

Delta Critical with Beta

Figure 2 shows the tendency of δ* with respect to β. The graph specifically shows delta critical ratios with respect to beta. Each line is a different value of gamma. The first trend it shows is that as beta increases, the delta critical values also increase. It is also shown that this trend is not linear. This is important because it shows that an increase in heat removal will have a greater increase in delta critical. Thus, a greater heat would be required to reach heat explosion.

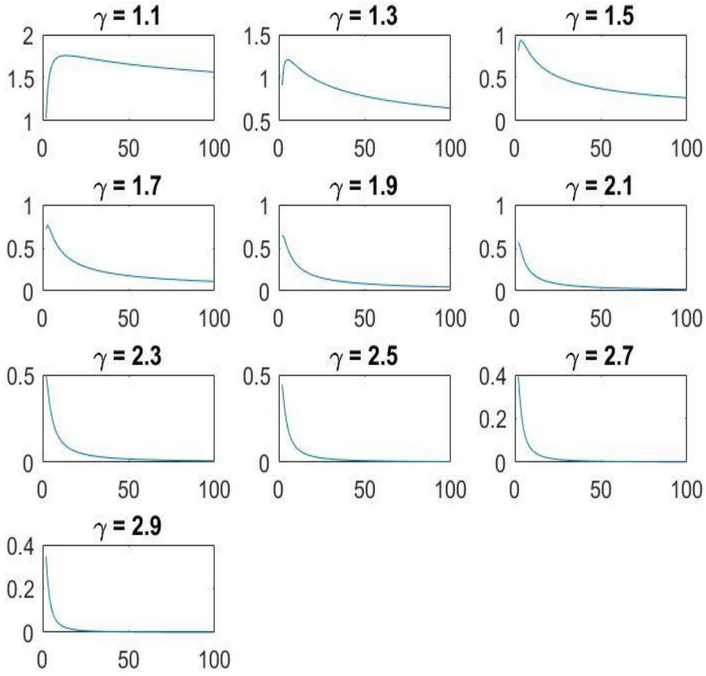

Fig. 1. δ values from Equation (1) verses T_m *on [1,100] at varying values of γ. The peak represents δ*.*

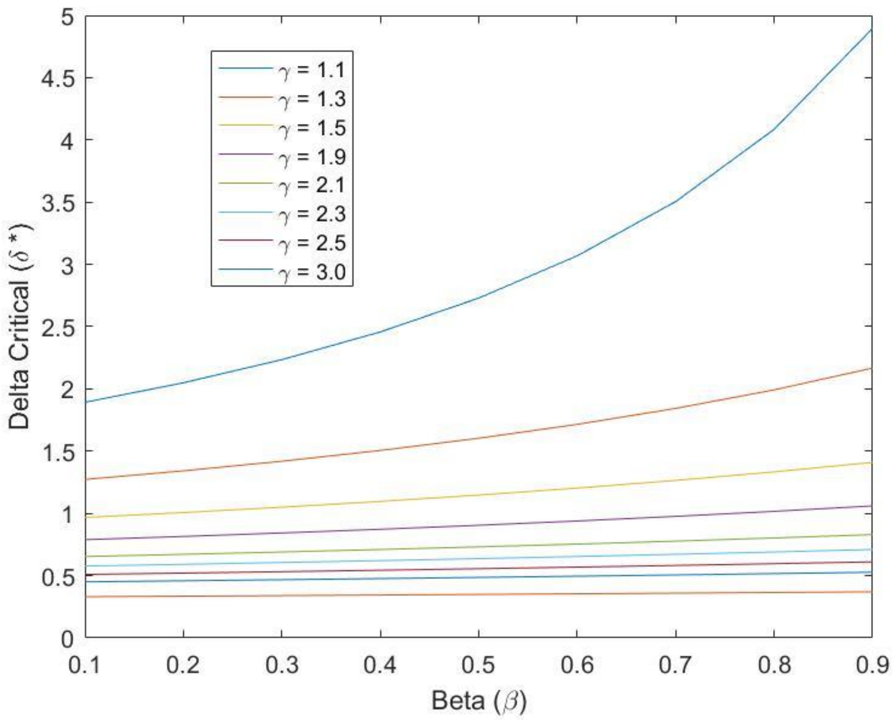

Fig. 2. δ^* ratios verses β for varying values of γ

Effect of Beta

Figure 3 shows that delta critical ratios are higher for high beta values and low gamma values. For larger values of gamma, beta has a small effect on these ratios. For smaller values of gamma, beta has a larger effect. This relationship works conversely for the effect of gamma. This suggests that the effect of heat dissipation affects the heat required for heat explosion more in materials with low heat retention, and less in materials with high heat retention.

Sensitivity Analysis

Figure 4 shows the results of the sensitivity analysis created using the results from **Figure 3** and Equations (3) and (4). Then, by taking the average of the partial derivatives over the whole interval in the gamma and beta position, it was found that on average gamma impacted the delta critical ratio about 2.29 times more than beta.

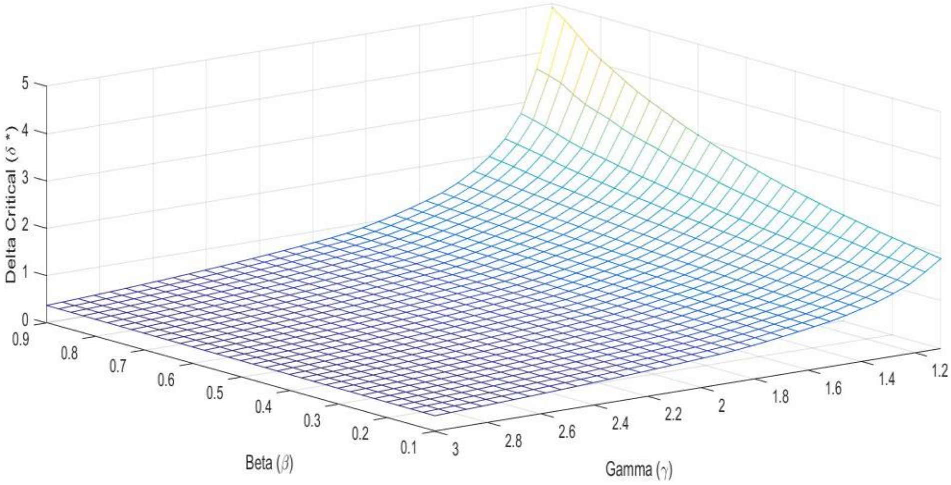

Fig. 3. 3-D model of delta critical ratios

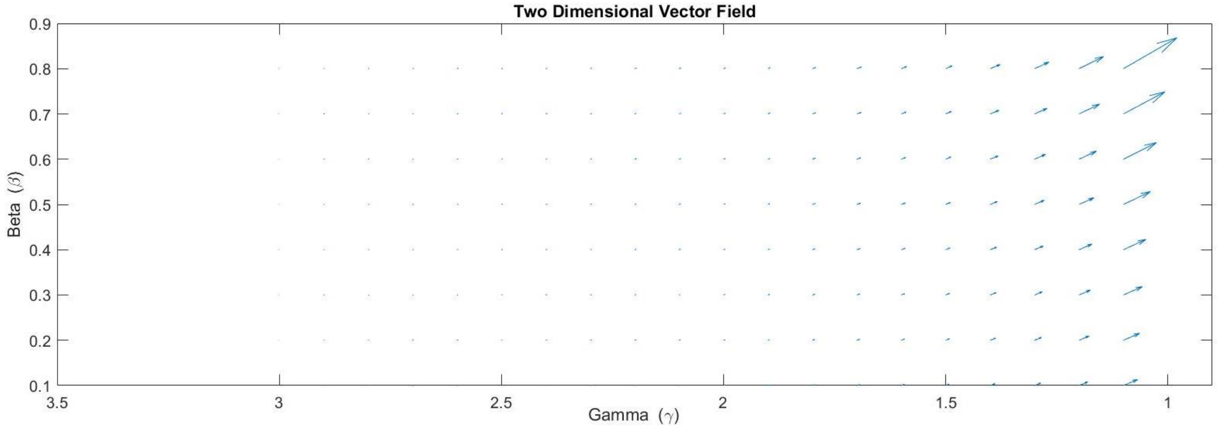

Fig. 4. Vector field obtained from partial derivatives

Summary. This paper revisits the study done in 2012 and presents a sensitivity analysis of the data. The model presented gives a way of modeling heat explosion based on temperature, heat removal and heat retention. The results of this modeling showed that it is important to consider these factors when designing materials that could be susceptible to heat explosion, and how these properties have an effect on the delta critical at which heat explosion occurs. The sensitivity analysis provided an understanding of which properties have the greatest effect on heat explosion.

An application where this information would be useful is in the use of prosthetics. Vibrations can occur in prosthetic limbs, causing them to heat. Knowing how different material properties affect heat explosion, prosthetics can be designed with built in cooling properties, or made from materials with less heat retention.

The goal of this model and sensitivity analysis is to further develop the understanding of thermal failure due to heat explosion in systems, and to determine which material properties have the greatest degree of effect on the occurrence of heat explosion. The model and analysis presented in this paper does accomplish this to some extent, but a further developed understanding can be produced in further research. The next step in doing this is to develop a model that predicts thermal failure based on Maxwell's laws, and compare it to the model based on Fourier's laws presented in this paper. In comparing the two models, it will be determined which model is more accurate and efficient in predicting heat explosion. Another future development of the model is to develop a physical meaning and dimensions for the material property parameters used. Accomplishing these goals will create a model of heat explosion that is accurate and consistent, and will allow engineers to predict thermal failure when designing systems.

Acknowledgements

We would like to give thanks to Clemson University. Specifically, the College of Engineering, Computing and Applied Sciences, and the School of Mathematical and Statistical Sciences. Without their resources, workspaces, software licensing and facility support, this research would not have been possible.

References

[1] D. Meinkohn, "Heat Explosion Theory and Vibrational Heating of Polymers," International Journal of Heat and Mass Transfer, Vol. 25, No. 4, 1981, pp. 645-648. Doi:10.1016/0017-9310(81)90008-9

[2] Viktorova, I., Scruggs, M., Zeller, I., & Fairchild, K. (2012). An Analysis of Heat Explosion for Thermally Insulated and Conducting Systems. Applied Mathematics, 03(06), 535-540. doi:10.4236/am.2012.36081

[3] I. Viktorova, J. V. Suvorova and A. E. Osokin, "Self Heating of Inelastic Composites under Cyclic Deformation," *Izvestiya AN USSR Mechanics of Solids*, Vol. 19, No. 1, 1984, pp. 99-105.

[4] I. Viktorova, "The Dependence of Heat Evaluation on Parameters of Cyclic Deformation Process," *Izvestiya AN USSR Mechanics of Solids*, No. 4, 1981, pp. 110-114.

Received on 12/12/2018

Published on 18/01/2019