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# REVIEWING HYDRO-MECHANICAL BASED NUMERICAL MODELS FOR CONSOLIDATION BEHAVIOR OF UNSATURATED SOILS

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**Keywords:** unsaturated soil mechanics, consolidation, flow law, elasticity.

**ABSTRACT.** Understanding the settlement behavior of different types of soil is important for almost every geotechnical structure. Many efforts have been done in the field of geotechnical engineering to evaluate and theorize the consolidation behavior of the soil in both saturated and dry conditions. However, similar efforts in the case of unsaturated soils are fairly limited. This research details the developed theories for the long-term deformation of unsaturated soils with a particular emphasis on the importance of flow law and elasticity at different phases of the soil. The state of the art in the field of Multiphysics consolidation is summarized in this paper. A series of the widely used mechanical concepts for unsaturated consolidation disintegrated in this study and elaborated in detail. This discretization of the current theories allowed further investigation of the pros and cons of each theory. A critical review of the literature assessed the applicability of the unsaturated consolidation theories in solving real-world problems.

**Introduction.** Consolidation is a process by which soils decrease in volume. According to Karl von Terzaghi [1] "consolidation is any process which involves a decrease in water content of saturated soil without replacement of water by air". However, this phenomenon could also happen in unsaturated soils so that the decrease in volume of the soil can be attributed to both air and water phases. The application of load over a soil specimen may result in the settlement [2-11] causing a change in the void structure and forces to act on the soil's water and air phases simultaneously and eventually results in an alteration in the water and air flow in a soil sample. American society for testing and materials define the consolidation as the increase in soil strength or density due to (1) effective stresses caused by soil water suctions, and (2) time whereby increases may occur at essentially constant values of suction. These two mechanisms of consolidation correspond respectively to primary and secondary consolidation as defined for a reduction in the volume of a soil mass [12-17] from an externally applied load. Fredlund and Rahardjo [18] explain the unsaturated consolidation process as the dissipation process of excess pore pressures including air pore pressure and water pore pressure as shown in Fig. 1. In general, reduction in volume is along with the expulsion of water and air under a time-dependent process [19-25]. As a result, the applied stress to soil can cause the particles being packed more tightly [26].

Interrelation of hydraulic and mechanical behaviors in unsaturated soils has been of interest in many geotechnical engineering-related problems in the last couple of decades [27-29]. During consolidation, excess pore-air and pore-water pressures are forced to dissipate through permeable boundaries. This dissipation process inevitably results in the reduction of the soil volume. This phenomenon can be mathematically described by inhomogeneous governing equations of flow according to Fick's (with respect to air phase) and Darcy's (with respect to water phase) laws (see Fig. 2).

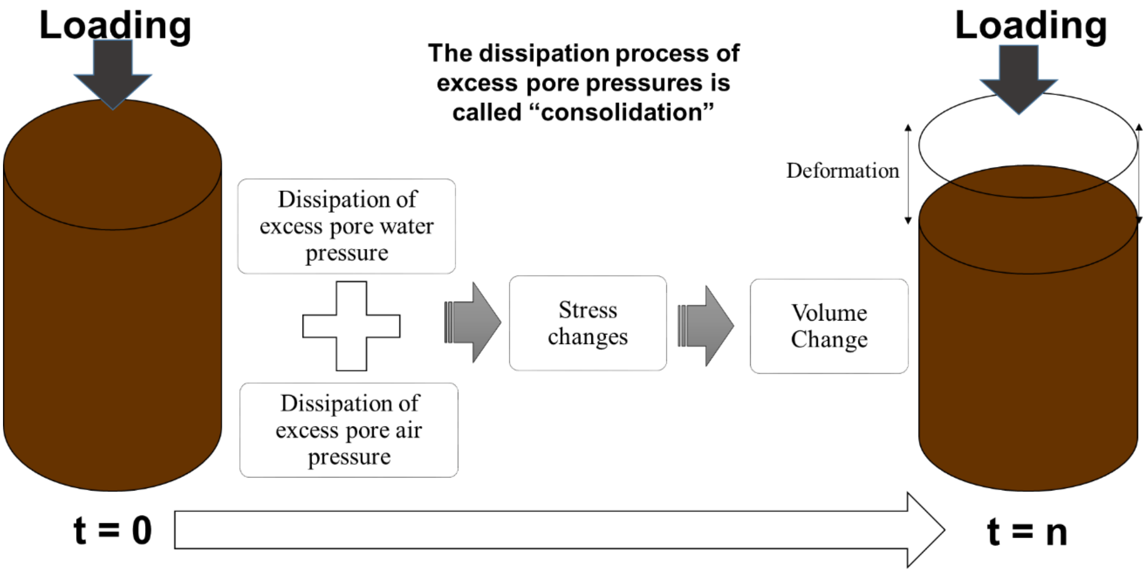

*Fig. 1. Unsaturated consolidation process for a soil sample.*

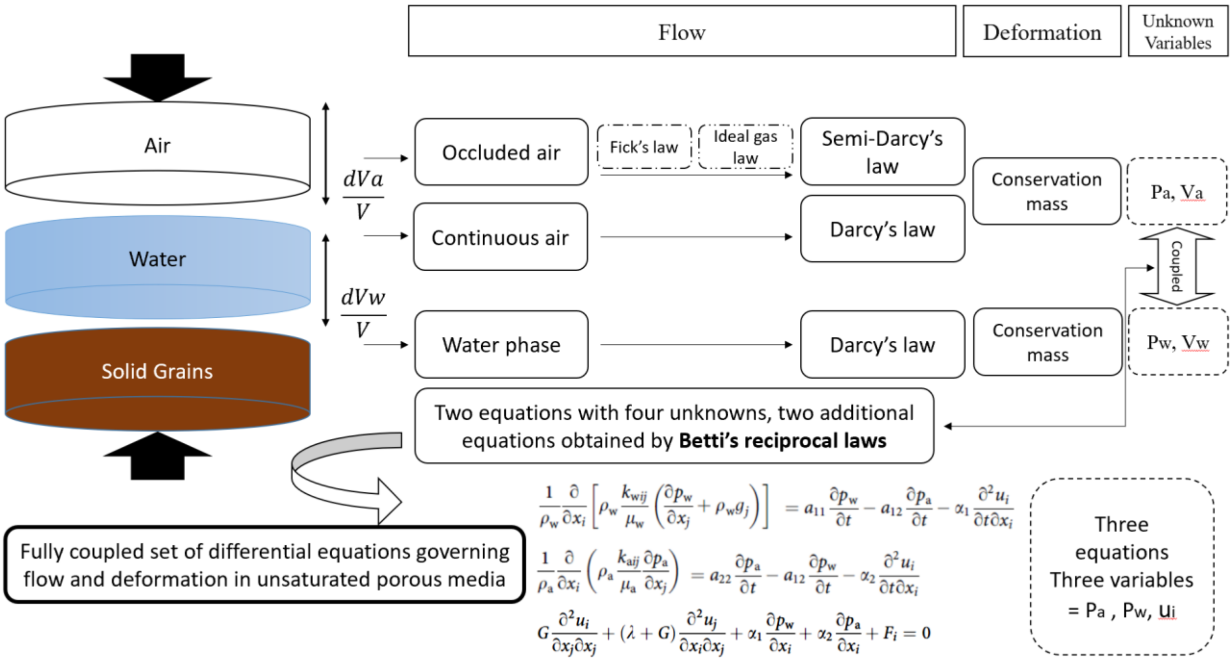

*Fig. 2. Fully coupled set of differential equations in unsaturated porous media.*

Indeed, consolidation theories for unsaturated conditions have been proposed by many researchers. Table 1 summarizes some of the proposed theories.

This review discusses Khalili's [30] theory as an example of the comprehensive theory in this subject. In addition, some critical points of view regarding this theory as well as many other theories in the field are discussed.

## Consolidation Theory.

Khalili's unsaturated consolidation theory [30] uses pore air, pore water, and matrix deformation vectors as the primary variables. In this set of equations, authors have used mechanical deformation and flow laws to derive the governing equation of consolidation. As for connecting these concepts, used effective stress approach have been used to define the relation of pore pressures and effective stress.

Theories of elasticity and total stress equilibrium used in deformation model and for flow model, Darcy's law, and conservation of mass have been used.

*Table 1. Summary of consolidation theories proposed by different researchers.*

| **Researcher** | **Year** |
|---|---|
| Blight | 1961 |
| Scott | 1963 |
| Liakopoulos | 1965 |
| Barden | 1965 |
| Fredlund and his coworkers | 1976-1993 |
| Lloret and Alonso | 1980 |
| B. A. Scsrefler and Zhan Xiaoyong | 1993 |
| N. Khalili, M. H. Khabbaz, S. Valliappan | 2000 |
| Zhang and Lytton | 2009 |
| Zhang and Li | 2011 |
| Blight | 1961 |

Macroscopic coupling between flow and deformation model defined by effective stress parameters. Moreover, the microscopic coupling between volume deformation of pore air and water fraction defined by using Betti's law.

**Deformation Model.**

This model is established on elasticity theory and the effective stress approach. First, we need to define the contribution of pore air and pore water pressure on the effective stress of the soil on unsaturated conditions. We use the effective stress approach for this aim as follows:

$$\sigma_{ij} = \sigma'_{ij} + \alpha_1 p_w \delta_{ij} + \alpha_2 P_a \delta_{ij}, \tag{1}$$

where $\alpha_1$ and $\alpha_2$ are effective stress parameters that define the effects and contribution of pore pressures on the effective stress. $\delta ij$ is the Kronecker's delta, which means these pressures only have effects on principal directions. Effective stress parameters in this theory are physically measurable parameters defined by considering an elemental volume of an unsaturated soil subjected to pore pressures and principal stresses [31]. The volumetric strain is also defined as:

$$\varepsilon_T = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \tag{2}$$

where $\varepsilon_1$ is defined for solid grains compression, $\varepsilon_1 = c_s\, p_w$ ($c_s$ is compressibility of solid grains, in many cases it ca,n be assumed as 0). $\varepsilon_2$ is defined for compression as a result of matric suction, $\varepsilon_2 = c_m\,(p_a - p_w)$ ($c_m$ is compressibility of soil with respect to change in matric suction). $\varepsilon_3$ is defined for drained compression of soil structure, $\varepsilon_3 = c\ (\sigma - p_a)$ ($c$ is drain compressibility of soil structure). It is noted that, for anisotropic conditions, fourth component of the strain might be available owing to deviatoric stress.

By comparing the above-mentioned equations with Equation 1, effective stress parameters would be calculated as:

$$\alpha_1 = \frac{c_m}{c} - \frac{c_s}{c}, \tag{3}$$

$$\alpha_2 = 1 - \frac{c_m}{c}, \tag{4}$$

By neglecting compression of solid grains ($c_s = 0$), $\alpha_1+\alpha_2 = 1$ and equation (1) become traditional form of Bishop effective stress $\sigma_{ij} = \sigma'_{ij} - p_a + \gamma\ (p_a - p_w)$

**Elasticity Theory.**

Now since we have effective stress parameters, we can use the elasticity theory and define deformation models in three-dimensional space. In these three equations, deformation, pore air, and water pressures are our primary variables and are unknown. In addition, we have Lame's constants as material parameters and effective stress parameters, as known variables:

$$(\lambda + G)\left(\frac{\partial^2 u_x}{\partial x \partial z} + \frac{\partial^2 u_y}{\partial y \partial z} + \frac{\partial^2 u_z}{\partial^2 z}\right) + G\left(\frac{\partial^2 u_z}{\partial^2 x} + \frac{\partial^2 u_z}{\partial^2 y} + \frac{\partial^2 u_z}{\partial^2 z}\right) + F_z + \alpha_1 \frac{\partial p_w}{\partial z} + \alpha_2 \frac{\partial p_a}{\partial z} = 0, \tag{5}$$

$$(\lambda + G)\left(\frac{\partial^2 u_x}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial y \partial z} + \frac{\partial^2 u_y}{\partial^2 y}\right) + G\left(\frac{\partial^2 u_y}{\partial^2 x} + \frac{\partial^2 u_y}{\partial^2 y} + \frac{\partial^2 u_y}{\partial^2 z}\right) + F_y + \alpha_1 \frac{\partial p_w}{\partial y} + \alpha_2 \frac{\partial p_a}{\partial y} = 0, \tag{6}$$

$$(\lambda + G)\left(\frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z} + \frac{\partial^2 u_x}{\partial^2 x}\right) + G\left(\frac{\partial^2 u_x}{\partial^2 x} + \frac{\partial^2 u_x}{\partial^2 y} + \frac{\partial^2 u_x}{\partial^2 z}\right) + F_x + \alpha_1 \frac{\partial p_w}{\partial x} + \alpha_2 \frac{\partial p_a}{\partial x} = 0, \tag{7}$$

Based on the background mentioned above, we have five unknown variables with only three equations, thus, more equations are required to find the unknowns.

**Flow Model.**

We are looking for equations related to pore water pressure to volumetric deformation in soil volume. As an acceptable approach, we can use Darcy's law for water phase and then mass conservation in the volume of the soil to find the relation of pore pressure and volumetric change.

Darcy's law is expressed as follows:

$$v_{wi} = \frac{k_{wij}}{\mu_w}\left(\frac{\partial p_w}{\partial x_j} + \rho_w g_j\right), \tag{8}$$

Considering the displacement of water as the integration of velocity with respect to time, it can be related to the conservation of mass. Which describes that displacement of water as inflow and outflow has a direct relationship with the total amount of water in the system. In this framework, it is known that in unsaturated soil condition inflow water is related to outflow and volumetric changes of water phase:

$$\frac{1}{\rho_w \partial x_i}\left[\rho_w \frac{k_{wij}}{\mu_w}\left(\frac{\partial p_w}{\partial x_j} + \rho_w g_j\right)\right] = n_w \mu c_w \frac{dp_w}{dt} - \frac{1}{v}\frac{dv_w}{dt}, \tag{9}$$

where $p_w$ and $V_w$ are two unknown variables in Equation 9. The other variables are all known. $n_w$ is volumetric water content, $c_w$ is water compressibility, $k_{wij}$ is the intrinsic permeability of the water phase, $\mu_w$ is dynamic viscosity of water phase, $\rho_w$ is the density of water.

**Air Phase.**

Based on the degree of saturation, the air phase condition can be different in a way that in the higher rate of saturation, the soil contains disconnected air bubbles whereas, in the lower rates, the continuous phase of air can be observed. Flow and its rate are depended on each of the above-mentioned conditions of air phase. For the occluded phase of air, we can use Fick's law and for the continuous phase of air, we can use Darcy's law to govern flow model in the air phase. Considering the first case and using Fick's law and gas law the following equation can be made:

$$\upsilon_{wi} = \frac{D^*_{ij}}{\mu_w}\frac{\partial p_a}{\partial x_j}, \tag{10}$$

Where $D^*_{ij}$ is the transmission coefficient of air phase and $\rho_a$ is air density. Moreover, by using Darcy's law for the second case the following equation is made:

$$\upsilon_{wi} = \frac{k_{aij}}{\mu_a}\frac{\partial p_a}{\partial x_j}, \tag{11}$$

where $k_{aij}$ is the intrinsic permeability of air phase and $\mu_a$ is air viscosity. Considering ($kaij = \mu_a D^*_{ij}/\rho_a$) and placing it in the equation it can be reached to an equation similar to the first case but with a different permeability coefficient. Henry law describes that at a constant temperature, the amount of a given gas that dissolves in a given type and volume of liquid is directly proportional to the partial pressure of that gas in equilibrium with that liquid. Using the concept of Henry law as a leakage term and air phase equations in conservation mass the following equation is obtained:

$$\frac{1}{\rho_a}\frac{\partial}{\partial x_i}\left[\rho_a \frac{k_{aij}}{\mu_a}\left(\frac{\partial p_a}{\partial x_j}\right)\right] = \frac{n_a}{p_a}\frac{dp_a}{dt} - \frac{1}{v}\frac{dv_a}{dt}, \tag{12}$$

Pore air pressure ($p_a$), and air volume ($V_a$) are the unknown variables in Equation 12. As mentioned before, we are looking for two more equations to find the unknowns in deformation equations.

In order to establish a link between pore water and pore air volumetric changes and the primary field variables, Betti's law can be used; which is based on work equilibrium. The law states that for a linear elastic material subject to two sets of forces P and Q, the work done by the set P through the displacements produced by the set Q is equal to the work done by the set Q through the displacements produced by the set P. It relates the works, which have been done by pore air, pore water, and isotropic pressures together.

we assume three different set of forces acting on the soil element. In case a) there is an external isotropic pressure of $d\sigma$, an internal pore water pressure of $dp_w$ and internal pore air pressure of $dp_a$ and in case b) we have internal and external pressure of $dp_w$, and in case of c) we have equal external and internal pore water pressure of $dp_w$ and a pore air pressure of zero. By using Betti's reciprocal law and for (a and b), (b and c) and (a and c) sets of parameters, we can derive an equation for volume strain of water and air in terms of $p_a$, $p_w$, and $\sigma$.

By using this theory and using mathematical efforts, we can reach to these two sets of the equation for three-dimensional space as:

$$\frac{1}{\rho_w}(\frac{\partial}{\partial x_i}\rho_w\left[\frac{k_{wxx}}{\mu_w}\left(\frac{\partial p_w}{\partial x}\right)+\frac{k_{wxy}}{\mu_w}\left(\frac{\partial p_w}{\partial y}\right)+\frac{k_{wxz}}{\mu_w}\left(\frac{\partial p_w}{\partial z}+\rho g\right)\right]+\frac{\partial}{\partial y}\rho_w\left[\frac{k_{wyx}}{\mu_w}\left(\frac{\partial p_w}{\partial x}\right)+\frac{k_{wyy}}{\mu_w}\left(\frac{\partial p_w}{\partial y}\right)+\frac{k_{wyz}}{\mu_w}\left(\frac{\partial p_w}{\partial z}+\rho g\right)\right]+\frac{\partial}{\partial z}\rho_w\left[\frac{k_{wzx}}{\mu_w}\left(\frac{\partial p_w}{\partial x}\right)+\frac{k_{wzy}}{\mu_w}\left(\frac{\partial p_w}{\partial y}\right)+\frac{k_{wzz}}{\mu_w}\left(\frac{\partial p_w}{\partial z}+\rho g\right)\right]=a_{11}\frac{\partial p_w}{\partial t}-a_{12}\frac{\partial p_w}{\partial t}-a_1(\frac{\partial^2 u_x}{\partial x\partial t}+\frac{\partial^2 u_y}{\partial y\partial t}+\frac{\partial^2 u_z}{\partial z\partial t}), \tag{13}$$

$$\frac{1}{\rho_a}(\frac{\partial}{\partial x}\rho_a\left[\frac{k_{wxx}}{\mu_w}\left(\frac{\partial p_a}{\partial x}\right)+\frac{k_{wxy}}{\mu_w}\left(\frac{\partial p_a}{\partial y}\right)+\frac{k_{wxz}}{\mu_w}\left(\frac{\partial p_a}{\partial z}\right)\right]+\frac{\partial}{\partial y}\rho_a\left[\frac{k_{wyx}}{\mu_w}\left(\frac{\partial p_a}{\partial x}\right)+\frac{k_{wyy}}{\mu_w}\left(\frac{\partial p_a}{\partial y}\right)+\frac{k_{wyz}}{\mu_w}\left(\frac{\partial p_a}{\partial z}\right)\right]+\frac{\partial}{\partial z}\rho_a\left[\frac{k_{wzx}}{\mu_w}\left(\frac{\partial p_a}{\partial x}\right)+\frac{k_{wzy}}{\mu_w}\left(\frac{\partial p_a}{\partial y}\right)+\frac{k_{wzz}}{\mu_w}\left(\frac{\partial p_a}{\partial z}\right)\right]=a_{22}\frac{\partial p_a}{\partial t}-a_{12}\frac{\partial p_a}{\partial t}-a_2(\frac{\partial^2 u_x}{\partial x\partial t}+\frac{\partial^2 u_y}{\partial y\partial t}+\frac{\partial^2 u_z}{\partial z\partial t}), \tag{14}$$

The unknown variables in Equations 13 and 14 are pore air and water pressures as well as deformation vectors. The other parameters are as:

$$a_{11}=n_w c_w+a_{12}, \tag{15}$$

$$a_{22}=\frac{n_a}{p_a+p_{atm}}+a_{12}, \tag{16}$$

$$c_m=\frac{dv/v}{d(p_a-p_w)}\ when\ d(\sigma-p_a)=0, \tag{17}$$

$$a_{12}=a_{12}=c'_m-a_1 c_m, \tag{18}$$

$$c'_m=\frac{dv_w/v}{d(p_a-p_w)}\ when\ d(\sigma-p_a)=0, \tag{19}$$

Equations 5-7,13 and 14 are the final equations for the consolidation theory. Through these equations, five unknowns including, three deformation components for three orthogonal direction, pore air, and pore water pressures exist. In order to solve these equations, we need to have both boundary condition because of the spatial derivation of equations and initial conditions because of the time derivation of equations.

**Parameter Identification.**

- λ, G: Lame's constant, both related to the drained modulus of elasticity and poison's ratio
- $K_{wij}$, $K_{aij}$: Unsaturated permeability tensors of the soil with respect to water and air respectively

It should be pointed out that there are plenty of equations in the literature, but authors suggested that we can use Brooks and Corey [32] equation to find a relation between relative permeability of air and water to saturation ratio (relative permeability is used to relate saturated permeability tensor to unsaturated permeability)

- $\mu_a$, $\mu_w$: Viscosity of air and water respectively
- $n$, $S_r$: porosity and saturation ratio of soil, measurable
- $C_w$, $C_s$, $C$, $C_m$, compressibility factors, measured through experiment and empirical equations
- $\alpha_1$, $\alpha_2$: Effective stress parameters relating pore-air and pore-water pressures to matrix deformation (are derived from Equations 3 and 4)
- $a_{11}$, $a_{22}$: Apparent compressibility of water and air respectively (are derived from 15 and 16)

- $a_{12}$: Coupling term relating microscopic pore-air and pore-water volumetric deformations (is derived from 18)

## Critical Points in Unsaturated Consolidation Theory and Conclusion.

### Summary.

1. In the Khalili's consolidation theory [30], although it was mentioned that the theory is based on effective stress approach, but in some cases such as using Nur and Byerlee [31] equations, and Betti's reciprocal laws, the matric suction was assumed to be an independent stress parameter as expressed in $\varepsilon_2 = c_m (p_a - p_w)$.

2. In Khalili's consolidation theory [30], it assumed that there are two constant effective stress parameters, which could be defined using conventional experiments. However, from the literature, it is clear that suction stress parameters depend on the moisture content, therefore conventional experiments should be modified in a way to be able to consider unsaturated conditions. In other words, the experiments should be done at the different desired ranges of moisture and suction. Moreover, moisture content and matric suction are not constant during the entire process and in each step, we have different pore pressures. Therefore, the assumption of the constant parameter is not valid in the entire experiment (see Equation1)

3. It seems that Equation 11 as expressed in the Khalili's paper is not defined properly. By assuming it as a typo, then it should be written as follow:

4.

$$\varepsilon_{ii} = c\sigma - \alpha_1 p_w - \alpha_2 p_a \text{ (Original)}$$

$$\varepsilon_{ii} = c\,(\sigma - \alpha_1 p_w - \alpha_2 p_a) \text{ (Corrected)}$$

5. In the derivation of equations, the compressibility factor, $c$, is not necessarily the same as the one in the equation, $\varepsilon_3 = c\,(\sigma - p_a)$. Since the compressibility factor in terms of change in isotropic external pressure is not always the same as the one with respect to effective stress.

6. In this theory as well as many other theories, the effects of deviatoric mechanical loading are neglected. However, in many cases, especially in realistic problems, we must face the anisotropic loading.

7. This theory defines two scenarios for the air flow in soil volume. However, this theory does not clearly state an exact mechanism to let the user know when the air phase is in occluded or continues form. The authors suggested using optimum water content as a criterion for this problem. However, this criterion needs more experimental validation for different types of soils such as clay, sand, and silt. Moreover, since during the consolidation the water content changes, how would be the transformation from one scenario to the other one, especially in numerically point of view. Generally, the effects of occluded air diffusion and its effects on a final settlement of the soil in unsaturated consolidation should be investigated thoroughly.

8. Many theories such as Khalili's consolidation theory [30] have used elasticity relation for the governing equations. Since experimental tests in literature showed that some material properties such as elastic modulus are moisture content dependent, is possible to neglect the variation of this term and its effects on consolidation equations? How much does it affect the result?

9. There are a few consolidation theories in the literature, which tries to define consolidation equations under the plastic condition and under a higher range of mechanical loading. Since, in reality, we have to face the conditions in which the soil deformation might not be in the elastic region, therefore, this subject can be a point of interest especially for practicing engineers.

10. The current theories are mainly valid in the capillary region since they use matric suction as their variable. These theories neglect the other physiochemical forces. This assumption may not thus work on the near residual water content conditions, especially for clayey soils.

11. In many uncoupled theories like Fredlund and Hassan [33] and Lloret and Alonso [34] equations, it was based on the assumption that air and water volumetric deformations are not interactive, and both are linearly related to the overall volumetric deformation of the soil. However, in the literature, it showed that this assumption is restrictive and is only valid for situations where the difference between the pore pressure in water and air is negligible.

12. In the uncoupled theories like Fredlund and Hassan [33] and Lloret and Alonso [34], they assumed that total stress doesn't change too much during the process and its derivation due to the time is zero. However, it seems that during consolidation as excess pore pressures dissipate and volume change, total stress would redistribute within the soil.

13. In many consolidation theories like in Khalili's theory [30], the application of unique soil water retention curve (SWRC) to define parameters like permeability ($k$) and compressibility ($C_m$), might not be an exact approach. Since the term of water content in SWRC has a volumetric based definition and the volume within two steps of consolidation change. It seems this effect is mainly neglected.

14. In Khalili's [30] theory as well as Fredlund and Hassan [33] theory, there are some parameters like compressibility factors that should be measured by using conventional tests like oedometer and triaxial test. However, these devices have their own deficiency (e.g., heterogeneous distribution of load, drainage path, and so on) and might not be reliable.

15. In Khalili's [30] theory assumed two different scenarios for air phase to flow in soil, one in occluded form and the other one in continues phase. However, for the water phase, it did not describe what would happen if the water phase is in a discontinuous form especially in lower water content.

16. When the soil is exposed to the changes in environmental loading, consolidation will occur. These changes in environmental loading are in cyclic form and it repeatedly goes through wetting and drying paths. Thus, the process of consolidation may repeat many times under this condition. Since we know that under this condition and as a result of volume change and hysteresis the material properties of soil changes, it can be a point of interest to investigate about the effects of hysteresis on consolidation theories.

17. It should be mentioned that consolidation can happen as a result of both changes, i.e., mechanical loading and environmental loading. When the soil is subjected to hydraulic, chemical, electrical, and thermal gradients, and it causes changes of water or air flow in soil volume or it changes the stress equilibrium, a consolidation problem can be faced.

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